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MODELING OF SUFFOSION OF WATER-BEARING STRATA

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A mathematical model of the sweeping-out of solid particles by a water flow from a porous stratum is suggested. The porous medium is represented in the form of two interpenetrating continua, one of which is connected with movable liquids and particles and the other with immovable ones. Corresponding expressions are obtained for dynamic porosity and permeability. In describing mass exchange between the continua, use is made of an ideal model of a porous medium in the form of a bundle of capillaries of different radii.

Flow of liquids with an admixture of solid particles is accompanied by their physicomechanical interaction. The particles can be stalled by a liquid from the walls of porous channels or, vice versa, settle on the walls; they are capable of sticking in the contractions of the pores, blocking a particle-containing liquid in the pore. The classical approach to modeling of this kind of phenomenon is based on the use of macroscopic conservation laws and kinetic relations for phases on the whole [1, 2]. The detailed structure of the pore space is not taken into account and the interaction of separate particles with the liquid and a porous skeleton is not considered. Kinetic constants have a very specific character, and solution of any problem begins with their selection. Investigations are known in which, to describe the processes of suffosion and colmatage, the size distribution function of the pores and model representation of a porous medium are used. In [3], the dependence of the rate of pore-channel convergence on the dimensions of a capillary, mean velocity of flow motion in it, and mean volume of particles is given. To evaluate the quantity of clogged pores, the probability approach was used. The change in the permeability was determined by means of an ideal model of a porous medium in the form of a bundle of capillaries.

Below, a mathematical model is suggested for the sweeping-out of solid particles by a water flow from a porous stratum. The porous medium is represented in the form of two interpenetrating continua, one of which is associated with movable liquids and particles and the other with immovable ones [4-6]. Corresponding expressions are obtained for dynamic porosity and permeability. In describing mass exchange between two continua, the ideal model of a porous medium was used in the form of a bundle of capillaries of different radii. The change in the mass of particles in the settled layer in each capillary and the change of the size distribution function of the pores is described by means of kinetic equations.

Mathematical Model. Let each point of a pure (without particles) water-bearing stratum be characterized by the following quantities: porosity $m = m(x, y, z)$ and absolute permeability $k^0 = k^0(x, y, z)$. Following [4], we arbitrarily split the porous medium into two interpenetrating continua characterized by the porosities m_1 and m_2:

$$m_1 + m_2 = m \,, \tag{1}$$

where $m_1 = m_1(x, y, z, t)$ is the dynamic porosity (the portion of the pore space occupied by the movable continuum) and $m_2 = m_2(x, y, z, t)$ is the portion of the pore space occupied by the immovable liquid and particles. In the second continuum we will separately consider two volumes: 1) the portion of the pore space in clogged and butt-end pores together with the liquid and particles contained in them; 2) the portion of the

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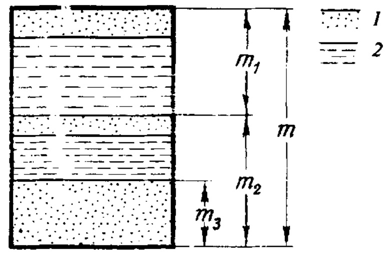

Fig. 1. Schematic splitting of pore space m: 1) particles; 2) water.

pore space $m_3 = m_3(x, y, z, t)$ occupied by deposited particles (Fig. 1). We neglect the inherent porosity of the deposited layer, i.e., we assume that the particles are densely packed and do not contain liquid in between.

We may assume that at the initial instant of time there are no clogged capillaries, and the second continuum is represented by an immovable water in the butt-end pores and by the particles deposited on the walls of the pore channels.

The conservation equations for the first continuum will be written in the form

$$\frac{\partial}{\partial t} m_1 + \operatorname{div} \mathbf{U} = -q, \tag{2}$$

$$\frac{\partial}{\partial t} (C_1 m_1) + \operatorname{div} (C_1 \mathbf{U} + D_u \operatorname{grad} C_1) = -q_c. \tag{3}$$

The equation of motion will be written in the form of the Darcy law:

$$\mathbf{U} = -\frac{k}{\mu} \operatorname{grad} (P), \tag{4}$$

where $k = k(x, y, z, t)$ is the permeability of the stratum with deposited particles that changes in time because of suffosion and clogging-up of a portion of the pore channels with particles.

The conservation equations for the second continuum are

$$\frac{\partial}{\partial t} m_2 = q, \tag{5}$$

$$\frac{\partial}{\partial t} (C_2 m_2) = q_c. \tag{6}$$

The concentration of particles in the stratum is connected with the concentrations of particles in the first and second continua by the obvious relation

$$Cm = C_1 m_1 + C_2 m_2. \tag{7}$$

In order to describe mass exchange between two continua and changes in the filtration-capacity characteristics of a porous medium, we will avail ourselves of the size distribution function $\varphi(r, t)$, which changes in time because of the sweeping-out of mass from the surface of the pore channels and because of the clogging-up of separate pore channels with particles. The following equation holds here [5]:

$$\frac{\partial \varphi}{\partial t} + u_r \frac{\partial \varphi}{\partial r} + u_\eta = 0, \tag{8}$$

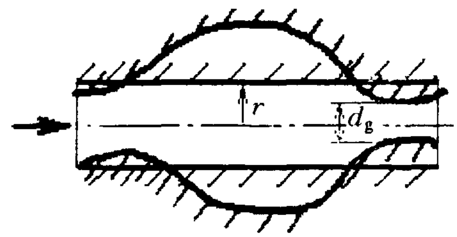

Fig. 2. Pore channel and a cylindrical capillary equivalent to it.

where u_r is the rate of change in the radii of the pore channels, which is determined by the process of suffosion; u_η is the rate of change in the quantity of the pore channels of radius r determined by the process of clogging-up of the pore channels.

For the initial time $t = 0$ it is necessary to know the size distribution of the pore channels:

$$\varphi(r, 0) = \varphi^0(r) . \quad (9)$$

However, only the size distribution of the pores $\varphi^*(r)$ is usually known for a pure stratum (i.e., the stratum without deposited particles) and the mass of deposited particles in a unit volume, and it is necessary to somehow redistribute the particles over separate pores, thus determining the function $\varphi^0(r)$. For this purpose, we will distribute the particles so that the following equality holds:

$$\varphi^0(r) = \varphi^*(\alpha r) , \quad \alpha \le 1 , \quad (10)$$

i.e., the relative change in the radii of the flow cross section of all the channels is the same due to the deposited particles. The magnitude of the parameter α will be found from the condition

$$m_3/m = \int_0^\infty r^3 (\varphi^* - \varphi^0)\, dr \Big/ \int_0^\infty r^3 \varphi^* dr . \quad (11)$$

Then the thickness of the deposited layer in a pore channel will be equal to

$$h = (1 - \alpha)\, r . \quad (12)$$

If the size distribution of the pore channels is assigned for the initial instant of time and the rates $u_r(t)$ and $u_\eta(t)$ are known, then for any subsequent instant of time the size distribution function of the pore channels will be determined by Eq. (8).

We note that the pores are clogged up only if the size of the particles is not smaller than the diameters of contractions (throats) d_g of the pore channels. Consequently, the rate u_η differs from 0 only in the region $0 \le d_g \le l$.

To evaluate the rate of expansion and clogging-up of the pore channels we will model a real porous medium by a system of cylindrical capillaries of different radii (Fig. 2) that have contractions of the pores [5, 6]. We will assume that (1) the particles in the liquid are distributed uniformly; (2) the ratio of the throat radius to the channel radius is the same for all the capillaries and is preserved in the process of tearing away of the particles from the walls of the channels; (3) a pore channel is blocked completely by a particle that came into the throat if the characteristic dimension of the particle is not smaller than the throat diameter.

We will consider a separate cylindrical capillary having radius r and length L. The deposited particles decrease the radius of the capillary to r^0 (Fig. 3).

To evaluate the rate of clogging of the pore channels we will use assumption (2), according to which $d_g = 2r\gamma$, and the maximum radius of the capillary that can be clogged is equal to $R = l/(2\gamma)$.

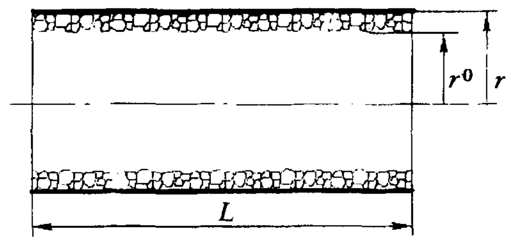

Fig. 3. Capillary with deposited particles.

Let u be the mean velocity of liquid mótion in the capillary. In what follows, we will assume that all the particles are of the same size and have a spherical shape. We will make use of the following two experimental facts:

1) the particles of a spherical shape of diameter l in a capillary of radius r can be torn away from the wall on attainment of a certain critical mean velocity [7, 8]:

$$u^* = u^* (r, l) ; \tag{13}$$

2) the intensity of suffosion is proportional to the difference between the mean velocity attained in the capillary and the critical velocity for each size of the particles.

In order to evaluate the critical velocity, it is necessary to write down the conditions of the equilibrium of forces acting on a particle [7, 8]. From the side of the moving liquid it is the hydrodynamic force that strives to involve the particle into motion. The particle is held on the channel wall by attraction forces due to friction and roughness of the surface.

We assume that the hydrodynamic force coincides in magnitude with the Stokesian force [9] at a mean velocity of the incoming flow equal to the mean velocity of liquid in a ring ($r^0 \geq r \geq r^0 + l$) and the holding force is proportional to the weight of the particle (similar to the friction force with a certain fictitious friction coefficient C_f). Then

$$6\pi\mu \frac{l}{2} u^* \left[1 - \left(1 - \frac{l}{r^0} \right)^2 \right] = C_f \frac{\pi}{6} l^3 \rho_p g , \tag{14}$$

whence

$$u^* = C_f \frac{l^2 \rho_p g}{18\mu \left[1 - \left(1 - \frac{l}{r^0} \right)^2 \right]} . \tag{15}$$

In accordance with the experimental data, the intensity of change in the volume of particles in the channels of radius r will occur according to the formula [1, 7]

$$\frac{\partial V_r}{\partial t} = \delta (u - u^*) V_r . \tag{16}$$

If N_r is the number of capillaries of radius r, then the volume of the deposited particles (accurate to h^2) in these capillaries is

$$V_r \cong 2\pi r h L N_r . \tag{17}$$

Then, from (16) it follows that

$$\frac{\partial h}{\partial t} = \delta\,(u - u^*)\,h \tag{18}$$

and the rate of change of the capillary radius because of suffosion is

$$u_r = \delta\,(u - u^*)\,h\,. \tag{19}$$

On the other hand, if the total volume of deposited particles per unit volume of the porous medium is equal to m_3, then the volume of the particles in the capillaries of radius r will be equal to

$$V_r = \frac{N\varphi 2\pi rhL}{N\int\limits_0^\infty \varphi 2\pi rhL dr}\,m_3 = \frac{rh\varphi}{\int\limits_0^\infty rh\varphi dr}\,m_3 \tag{20}$$

and the intensity of setting particles into motion q_c^r because of suffosion per unit volume of the porous medium will be determined according to the formula

$$q_c^r = \delta\int\limits_0^\infty (u - u^*)\frac{rh\varphi}{\int\limits_0^\infty rh\varphi dr}\,m_3 dr = \delta m_3\,\frac{\int\limits_0^\infty (u - u^*)\,rh\varphi dr}{\int\limits_0^\infty rh\varphi dr}\,. \tag{21}$$

The mean velocity in a pore channel u is related to the filtration rate $\mathbf{U}$ by the relation

$$u = |\mathbf{U}|\,r^2/(8k)\,, \tag{22}$$

which can easily be obtained by combining the Poiseuille law for a capillary with the Darcy law for a porous-medium element represented by a bundle of capillaries.

In the time Δt the radii of the capillaries change because of suffosion by the magnitude

$$\Delta r = u_r\,\Delta t\,, \tag{23}$$

which leads to an increase in clearance. The new clearance (and consequently porosity) is

$$m_1\,(t + \Delta t) = m_1\int\limits_0^\infty \varphi\,(r + \Delta r)^2\,dr \Big/ \int\limits_0^\infty \varphi r^2 dr \tag{24}$$

or, neglecting the term that contains $(\Delta r)^2$ and taking into account (23), we obtain

$$m_1\,(t + \Delta t) = m_1\int\limits_0^\infty \varphi\,(r + 2u_r r\Delta t)dr \Big/ \int\limits_0^\infty \varphi r^2 dr\,, \tag{25}$$

i.e., the clearance will change by

$$\Delta m_1 = 2m_1\int\limits_0^\infty \varphi u_r r\Delta t dr \Big/ \int\limits_0^\infty \varphi r^2 dr\,. \tag{26}$$

The quantity m_3 changes by the same magnitude. Having divided (26) by Δt and let Δt tend to zero, for m_3 we have

$$\frac{\partial m_3}{\partial t} = 2m_1 \int_0^{\infty} \varphi u_r r dr \Big/ \int_0^{\infty} \varphi r^2 dr . \tag{27}$$

Let us consider the channels with radii of throats that satisfy the clogging condition $d_g \le 1$.

We will assume that the fraction of clogged capillaries whose radii satisfy the clogging condition is proportional to the number of particles that occurred in these channels with the proportionality factor β $(0 < \beta \le 1)$.

In the time Δt, into the capillaries of radius r will enter the particles whose volume is proportional to the flux of liquid in these pores:

$$n_r = \frac{C_1 u \Delta t}{\Omega} N_r = r^2 N_r \frac{C_1 |\mathbf{U}|}{8k\Omega} \Delta t . \tag{28}$$

The quantity of the clogged capillaries is equal to βn_r.

The change of the size distribution function of the pores due to the clogging in the time Δt can be calculated as

$$\Delta \eta = -\beta n_r / N = -\beta r^2 \eta \frac{C_1 |\mathbf{U}|}{8k\Omega} \Delta t , \tag{29}$$

and the rate u_η will be equal to

$$u_\eta = -\frac{\Delta \eta}{\Delta t} = \beta r^2 \eta \frac{C_1 |\mathbf{U}|}{8k\Omega} . \tag{30}$$

Thus, the coefficients u_r and u_η of Eq. (8) are determined by the dependences

$$u_r = \delta (u - u^*) h , \tag{31}$$

$$u_\eta = \begin{cases} \beta r^2 \eta C_1 |\mathbf{U}| / (8k\Omega) & (2r \le l/\gamma) , \\ 0 & (2r > l/\gamma) . \end{cases} \tag{32}$$

The change in the absolute permeability caused by the change in the structure of the pore space because of suffosion will be evaluated by representing the permeability for the current moment $k(x, y, z, t)$ in the form of the product

$$k = \bar{k} k^0 , \tag{33}$$

where the coefficient $\bar{k}(x, y, z, t)$ that characterizes the relative change in the permeability of the first medium will be calculated using the model of parallel capillaries and the Poiseuille law:

$$\bar{k} = \int_0^{\infty} r^4 \varphi (r)\, dr \Big/ \int_0^{\infty} r^4 \varphi^* (r)\, dr . \tag{34}$$

The intensity of the transition of the liquid from the movable to the immovable state is determined by the volume of clogged capillaries and can be calculated from the formula

$$q = \pi \beta C_1 \frac{|\mathbf{U}|}{\Omega} \int_0^R \varphi(r) \, r^4 dr \Big/ \int_0^\infty \varphi(r) \, r^2 dr \tag{35}$$

or

$$q = m_1 \int_0^R u_\eta \, r^2 dr \Big/ \int_0^\infty \varphi(r) \, r^2 dr \, . \tag{36}$$

The intensity of the transition of the particles to an immovable state because of clogging q_c^{v} is

$$q_c^{\eta} = C_1 q \, , \tag{37}$$

and the total intensity of the transition of the particles to the immovable state is

$$q_c = q_c^{\eta} - q_c^{r} \, . \tag{38}$$

NOTATION

m, porosity; D_u, coefficient of convective diffusion; $\mathbf{U}$, filtration rate; P, pressure; k^0, absolute permeability of the stratum without particles; μ, dynamic viscosity of the liquid; ρ, density of the particle; C, volume concentration of solid particles; C_1, volume concentration of particles in the first continuum; C_2, volume concentration of the particles in the second continuum; r, radius of the pore channel; d_g, diameter of the throat of the pore channel; γ, constant equal to the ratio of the throat radius to the pore-channel radius; t, time; η, fraction of the capillaries of radius r; l, characteristic dimension of the particles; L, characteristic length of the pore channels; Ω, mean volume of one particle; N, total quantity of the capillaries in a sample with a unit cross-section area; N_r, number of capillaries of radius r; n_r, number of particles occurring in the capillaries of radius r; q, intensity of liquid transition from the movable to the immovable state; g, free-fall acceleration; δ, kinematic constant. Subscripts: c, concentration; 0, initial value; p, particle.

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